

Vector field as a quintessence partner

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Abstract. We derive generic equations for a vector field driving the evolution of flat homogeneous isotropic universe and give a comparison with a scalar field dynamics in the cosmology. Two exact solutions are shown as examples, which can serve to describe an inflation and a slow falling down of dynamical “cosmological constant” like it is given by the scalar quintessence. An attractive feature of vector field description is a generation of “induced mass” proportional to a Hubble constant, which results in a dynamical suppression of actual cosmological constant during the evolution.

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1. Introduction

A scalar field constitutes a basic ingredient in a preferable theoretical treatment of two phenomena recently established experimentally by astronomical observations [1, 2]: an inflation expansion of the Universe and a dark energy pre-dominance at the current time.

The inflation [3] driven by a slowly rolling scalar field gives correct initial conditions for inhomogeneous perturbations of a matter density, which cause the observed anisotropy in the cosmic microwave background [2]. This stage of scalar field contribution takes place at early times before a Big Bang, so that a Harrison–Zel’dovich spectrum of density inhomogeneity is formed.

At present, the accelerated expansion of Universe [1] is caused by a matter with a negative pressure, a quintessence [4, 5, 6], which is modelled by a scalar field with an appropriate potential, so that the visible “cosmological term” acquires a dynamics, that can explain an unnatural value of “dark energy” scale. A scalar phantoms with a negative kinetic energy could effectively be involved in such the studies, too [7].

There are some attempts to ascribe a dark matter in halos of galaxies [8, 9, 10] to a scalar field, too, though properties of such the fields at the galaxy scales should be rather different from those of quintessence at the cosmic scale [11, 12, 13].

Therefore, scalar fields essentially contribute to the modern understanding of cosmology and take control of its theoretical progress. The only problem is an arbitrariness in a choice of scalar potentials, which can be somehow motivated but not certainly derived in an explicit form.

In the present paper we consider the expansion of homogeneous isotropic universe in the presence of a vector field, which could be a partner of quintessence. In contrast to both ref. [14], wherein a gauge vector field with a global symmetry was investigated, and ref. [15], where a non-linear electrodynamics led to an acceleration of universe, we study a simple Lorentz-invariant form of lagrangian for a vector field interacting gravitationally only,

$$\mathcal{L}_V = \xi \frac{1}{2} g^{\mu\nu} (\nabla_\mu \phi^m) (\nabla_\nu \phi^n) g_{mn} - V(\phi^2), \quad (1)$$

where ξ is a vector field signature[‡], that can be normal ($\xi = -1$) or phantom ($\xi = +1$), respectively[§]. A motivation for the lagrangian is twofold. First, a scalar phantom with a negative sign of kinetic energy recently studied in the context of cosmology [6, 17] could be replaced with a time-component of vector field in the normal mode of the signature. Second, any gauge vector field, for example, the abelian field \mathcal{A}_μ has a purely gauge component $\mathcal{A}_\mu^G = \partial_\mu \omega(x)$, which does not interact with a matter, at all. So, in the field theory the gauge invariance preserves that the purely gauge component has a bare free propagator invisible for the matter detectors. Moreover, this propagator does provide the negative sign of kinetic energy. However, the propagator could become gauge-dependent. Nevertheless, it induces a gravitational force due to a contribution to the energy-momentum tensor. The influence of this force by the phantom component of vector field in a curved space-time is under question. Thus, two mentioned items are addressed in the present paper.

Fixing the Friedmann–Robertson–Walker metric

$$ds^2 = dt^2 - a^2(t) [dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2], \quad (2)$$

we develop a variational technique simplifying a getting the motion equations in section 2 and then derive the evolution equations for the scale factor $a(t)$ and the vector field, which acquires an “induced mass” term with a characteristic scale of Hubble constant^{||}. In section 3 two exact solutions are shown. The first solution gives a constant field due to a compensation of potential variation by the induced mass term, that can be possible for a specific potential only, so that de Sitter space-time regime takes place. The second solution is a free (zero potential) vector field evolving in the presence of an actual vacuum energy, i.e. the cosmological term. However, the contribution of this cosmological constant to the Hubble constant gets a suppression during the evolution, so that any value can be suppressed down to the observed scale. A slow-roll approximation and an interaction of vector field with a scalar quintessence are also described. The results are summarized in Conclusion.

2. Generic equations

In this section we develop a variational method to derive the evolution equations in the case of FRW metric of (2). So, we give explicit expressions for the Christoffel symbols and curvature in terms of evolving scale factor $a(t)$. A usual variation of action over

[‡] The signature of metric is assigned to $(+, -, -, -)$.

[§] Note that the four-vector is generically composed by spin-1 and spin-0 components (see ref. [16] for discussion on a covariant object decomposition into components with definite spins). So, such the companion component can cause a problem with unitarity, which we do not consider here. However, we would implicitly suggest, that the modes, which could non-gravitationally interact with a matter, conserve the unitarity.

^{||} Similar effect for interacting scalar fields was considered in [18].

$a(t)$ results in the motion equations. However, we show that a complete set of equations can be obtained, if we add the invariance under the scale transformation of time. The reason for such the procedure is quite transparent, since the fixing of FRW metric excludes the variation of time-time component of the metric, that can be recovered by the variation of time scale. We check this approach by getting well-known equations for the scalar field in the FRW background. Further, the same technique is applied to derive the evolution equations in the case of vector field, that is significantly simpler than a straightforward calculation of double covariant derivatives.

2.1. Metric values

The interval of (2) corresponds to the metric

$$g_{tt} = 1, \quad g_{ij} = -a^2(t) \gamma_{ij}, \quad i, j = r, \theta, \phi, \quad (3)$$

with the following non-zero diagonal elements:

$$\gamma_{rr} = 1, \quad \gamma_{\theta\theta} = r^2, \quad \gamma_{\phi\phi} = r^2 \sin^2 \theta. \quad (4)$$

The Christoffel symbols calculated by a general definition

$$\Gamma_{\mu\nu}^\lambda = \frac{1}{2} g^{\lambda\rho} (\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu}) \quad (5)$$

are given by¶

$$\begin{aligned} \Gamma_{ij}^t &= \dot{a} a \gamma_{ij}, & \Gamma_{tj}^i &= \frac{\dot{a}}{a} \delta_j^i, & \Gamma_{r\theta}^\theta &= \Gamma_{r\phi}^\phi = \frac{1}{r}, \\ \Gamma_{\theta\theta}^r &= -r, & \Gamma_{\phi\phi}^r &= -r \sin^2 \theta, & \Gamma_{\phi\phi}^\theta &= -\sin \theta \cos \theta, \\ \Gamma_{\theta\phi}^\phi &= \frac{\cos \theta}{\sin \theta}, \end{aligned} \quad (6)$$

while the symbols are symmetric over the contra-variant indices, and other symbols not listed above are equal to zero. In eqs.(6) we do not explicitly show the dependence of scale factor on the time.

Then, the non-zero elements of Ricci tensor are the followings:

$$R_{tt} = -3 \frac{\ddot{a}}{a}, \quad R_{ij} = (\ddot{a} a + 2\dot{a}^2) \gamma_{ij}, \quad (7)$$

while the scalar curvature is equal to

$$R = -6 \left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 \right]. \quad (8)$$

Therefor, the Einstein–Hilbert action of gravity

$$S_G = -\frac{1}{16\pi G} \int R \sqrt{-g} d^4x = \frac{3}{8\pi G} \int (\ddot{a} a^2 + \dot{a}^2 a) d^4x$$

after the integration by parts for the first term, takes the form

$$S_G = -\frac{3}{8\pi G} \int \dot{a}^2 a d^4x, \quad (9)$$

while the surface terms are not relevant to the variational equations at fixed values of dynamical variables at the surface.

¶ We use an ordinary notation for the time-derivative by the dot-over symbol $\partial_t f(t) = \dot{f}(t)$.

2.2. Scalar field

In the FRW metric, the action of scalar field $\phi(t)$ depending on the time, only, is

$$S_S = \int a^3 \left(\frac{1}{2} \dot{\phi}^2 - V_S(\phi) \right) d^4x. \quad (10)$$

The Euler–Lagrange equation of motion

$$\frac{\partial \mathfrak{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathfrak{L}}{\partial (\partial_\mu \phi)} = 0$$

with the density of lagrangian $\mathfrak{L} = \sqrt{-g} \mathcal{L}$, straightforwardly gives

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V_S}{\partial \phi} = 0, \quad (11)$$

where we define the Hubble constant $H = \dot{a}/a$.

Analogously, the Lagrange equation obtained by the variation over the scale factor $a(t)$ including the gravity and scalar matter actions, results in

$$-\frac{3}{8\pi G} \left[\dot{a}^2 - \frac{d}{dt}(2\dot{a}a) \right] + 3a^2 \left(\frac{1}{2} \dot{\phi}^2 - V_S(\phi) \right) = 0,$$

Hence, we arrive at the following equation determining the second derivative of scale factor $a(t)$:

$$2\frac{\ddot{a}}{a} = -H^2 - 8\pi G \left(\frac{1}{2} \dot{\phi}^2 - V_S(\phi) \right). \quad (12)$$

A key moment is a variation of the time scale, which is defined by the following infinitesimal transformations:

$$\delta_\lambda dt = -\delta\lambda dt, \quad \delta_\lambda \dot{a} = \delta\lambda \dot{a}, \quad \delta_\lambda \dot{\phi} = \delta\lambda \dot{\phi}, \quad (13)$$

that results in the equation

$$H^2 = \frac{8\pi G}{3} \left(\frac{1}{2} \dot{\phi}^2 + V_S(\phi) \right). \quad (14)$$

Then, the substitution of H^2 in (12) by (14) gives a usual evolution equation of scale factor governed by the scalar field

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3} \left(\dot{\phi}^2 - V_S(\phi) \right). \quad (15)$$

Let us compare the above method with the standard procedure giving the field equations of gravity, i.e. the equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad (16)$$

where the energy-momentum tensor of scalar field is given by

$$T_{\mu\nu} = (\partial_\mu \phi) (\partial_\nu \phi) - \mathcal{L} g_{\mu\nu},$$

so that the time components give the energy density of the scalar field

$$\rho_S = \frac{1}{2} \dot{\phi}^2 + V_S(\phi),$$

while the remaining ones determine its pressure

$$p_S = \frac{1}{2} \dot{\phi}^2 - V_S(\phi).$$

Therefore, we get ordinary evolution equations of the form

$$H^2 = \frac{8\pi G}{3} \rho, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p), \quad (17)$$

whereas the first equation is the positive energy condition obtained by the variation of time component of the metric. Let us stress that the same equation we have got under the invariance over the time dilation.

Finally, note, that (15) is a consequence of two other equations: (11) and (14).

2.3. Vector field

Let us try the same method to get the equations for the gravity and vector field.

The covariant derivative of the vector field ϕ^m

$$\nabla_\mu \phi^m = \partial_\mu \phi^m + \Gamma_{\mu n}^m \phi^n \equiv \phi^m_{;\mu} \quad (18)$$

is reduced to the following non-zero components:

$$\phi^t_{;t} = \dot{\phi}_0, \quad \phi^i_{;j} = \delta_j^i \frac{\dot{a}}{a} \phi_0, \quad (19)$$

where we suppose an isotropic homogeneous solution for the vector field

$$\phi^m = \{\phi_0(t), \mathbf{0}\}$$

in the FRW background. Then the action of vector field is equal to

$$S_V = \int a^3 \left(\xi \frac{1}{2} \dot{\phi}_0^2 + \xi \frac{3}{2} H^2 \phi_0^2 - V(\phi_0^2) \right) d^4x. \quad (20)$$

Therefore, the Lagrange equation for the vector field ($\delta S_V / \delta \phi_0 = 0$) reads off

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 - 3H^2\phi_0 + \xi \frac{\partial V}{\partial \phi_0} = 0. \quad (21)$$

As was for the scalar field, the Hubble constant generates a friction $\dot{\phi}_0$ -term in the equations of motions. In addition, the vector field interacting with the FRW metric possesses a feature caused by the induced mass term in (21),

$$m_{\text{ind}}^2 = -3H^2,$$

which has a negative sign and depends on the dynamical value of H . In the normal mode, $\xi = -1$, for the spatial vector-field ϕ , the induced potential determines a positive energy of ϕ_0 , while the kinetic term of time-component ϕ_0 has a phantom sign. Nevertheless, we continue the consideration for both signatures.

Further, the variation over the scale factor gives

$$\begin{aligned} 2 \frac{\ddot{a}}{a} (1 - 4\pi G \xi \phi_0^2) + H^2 (1 - 4\pi G \xi \phi_0^2) \\ + 8\pi G \left(\xi \frac{1}{2} \dot{\phi}_0^2 - V - 2\xi H \phi_0 \dot{\phi}_0 \right) = 0, \end{aligned} \quad (22)$$

which is more complex than that of the scalar field.

Finally, extending the time-dilation of (13) by an additional trivial condition for the vector field

$$\delta_\lambda \phi_0 = 0, \quad (23)$$

implying that the metric is not varied, we derive the third equation (the positive energy condition),

$$H^2 = \frac{8\pi G}{3} \left(\xi \frac{1}{2} \dot{\phi}_0^2 + V + \xi \frac{3}{2} H^2 \phi_0^2 \right) \quad \Leftrightarrow \quad (24)$$

$$H^2 = \frac{8\pi G}{3} \frac{1}{1 - 4\pi G \xi \phi_0^2} \left(\xi \frac{1}{2} \dot{\phi}_0^2 + V \right). \quad (25)$$

Again, the form of (24) contains the “mass term” induced by the Hubble constant. Its contribution can be rewritten in (25), wherein we can observe an effective gravitational constant depending on the dynamical field,

$$G_{\text{eff}} = G \frac{1}{1 - 4\pi G \xi \phi_0^2}. \quad (26)$$

Three equations of (21), (22) and (25) are not independent. Indeed, (22) is a straightforward consequence of (21) and (25), which gives a good check of validity for the procedure used.

Explicitly, we rewrite (22) in a more spectacular form,

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3} \frac{1}{1 - 4\pi G \xi \phi_0^2} (-\xi \dot{\phi}_0^2 + V + 3\xi H \phi_0 \dot{\phi}_0), \quad (27)$$

which is analogous to the case of scalar field in (14) and (15), if only one substitutes the effective gravitational constant of (26) and introduces an induced pressure

$$\Delta p_{\text{ind}} = -2\xi H \phi_0 \dot{\phi}_0, \quad (28)$$

which is positive, if the signature is normal $\xi = -1$, the universe is expanded $H > 0$, and the field squared is growing up, $\partial_t(\phi_0^2) > 0$. Then, common equations of (17) with the effective gravitational constant are reproduced.

Finally, we give general equations for the evolution of flat homogeneous isotropic universe in the presence of vector field and a matter with the energy density ρ_m and pressure p_m ,

$$H^2 = \frac{8\pi G_{\text{eff}}}{3} \rho, \quad (29)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_{\text{eff}}}{3} (\rho + 3p), \quad (30)$$

where

$$\rho = \rho_m + \xi \frac{1}{2} \dot{\phi}_0^2 + V, \quad (31)$$

$$p = p_m + \xi \frac{1}{2} \dot{\phi}_0^2 - V - 2\xi H \phi_0 \dot{\phi}_0. \quad (32)$$

A complete set of equations includes (21), of course. Then, one can easily check that the conservation law for the matter is valid as a consequence of (21), (29)–(32):

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0. \quad (33)$$

Thus, the induced mass term of the vector field in the FRW metric provides a rich phenomenology, two examples of which are shown in the next section.

3. Some solutions

The consideration of evolution is essentially simplified, if we neglect the external matter.

3.1. Constant field

First, let us describe the case of constant field, i.e. we put

$$\dot{\phi}_0 \equiv 0 \quad \Rightarrow \quad \dot{G}_{\text{eff}} = 0.$$

Then we get

$$3H^2\phi_0 = \xi \frac{\partial V}{\partial \phi_0}, \quad (34)$$

and

$$H^2 = \frac{8\pi G}{3} \frac{1}{1 - 4\pi G \xi \phi_0^2} V, \quad (35)$$

so that the potential should have a form

$$V = V_0 \frac{1}{1 - 4\pi G \xi \phi_0^2}. \quad (36)$$

Then, the Hubble constant is independent of time,

$$\dot{H} \equiv 0,$$

and its square is positive, if $V_0 > 0$,

$$H^2 = \frac{8\pi G_{\text{eff}}^2}{3G} V_0.$$

At $\xi = -1$ we avoid a singularity in the potential. In this case an actual value of primary cosmological constant can be suppressed as

$$V_0^{\text{eff}} = V_0 \frac{1}{(1 + 4\pi G \phi_0^2)^2},$$

if the field value is large in comparison with the Planck mass, $m_{\text{Pl}}^2 = 1/4\pi G$.

3.2. Cosmological constant

Second, we make a field potential to be a trivial constant

$$V \equiv V_0.$$

Then, the field could asymptotically evolve with a zero acceleration

$$\ddot{\phi}_0 \equiv 0, \quad \text{at } \phi_0^2 \gg m_{\text{Pl}}^2.$$

Indeed, we have got

$$\begin{aligned} \dot{\phi}_0 = H \phi_0 &\Rightarrow \\ H^2 \left(1 - \xi \frac{\phi_0^2}{m_{\text{Pl}}^2} \right) &= \frac{2}{3m_{\text{Pl}}^2} \left(V_0 + \xi \frac{1}{2} H^2 \phi_0^2 \right) \Rightarrow \\ H^2 &\approx -\xi \frac{V_0}{2\phi_0^2}. \end{aligned}$$

Moreover, the acceleration parameter is

$$\frac{\ddot{a}}{a} \frac{1}{H^2} \equiv q = \frac{1}{1 - \xi \phi_0^2/m_{\text{Pl}}^2} \approx 0.$$

The asymptotic solution at $\phi_0^2 \gg m_{\text{Pl}}^2$ reads off

$$\phi_0(t) = (t - t_*) \sqrt{-\xi \frac{V_0}{2}}, \quad (37)$$

where t_* is an integration constant. Then, we find

$$H(t - t_*) = 1, \quad (38)$$

and the acceleration is equal to zero.

No doubt, the above two solutions can be considered as preliminaries for a phenomenological treatment, when one should include the matter and radiation as well as a scalar quintessence or inflation field. Nevertheless, we see that the vector field could serve as an original source for both regimes: the inflation and current acceleration in the universe expansion.

3.3. Slow-roll approximation

Consider the case, when one can approximately neglect higher derivative terms for the vector field, i.e. the kinetic energy in comparison to the potential one and a variation of the kinetic energy in the field equations. So, we get the following slow-roll conditions:

$$\epsilon_V = \left| \frac{\dot{\phi}_0^2}{2V} \right| \ll 1, \quad \eta_V = \left| \frac{\ddot{\phi}_0}{3H\dot{\phi}_0} \right| \ll 1.$$

Then, the field equations are reduced to

$$H^2 = \frac{2}{3m_{\text{Pl}}^2} \frac{V}{1 - \xi \phi_0^2/m_{\text{Pl}}^2}, \quad 3H\dot{\phi}_0 - 3H^2\phi_0 + \xi V' = 0,$$

where $V' = \partial V / \partial \phi_0$. Then

$$\begin{aligned} \epsilon_V &= \frac{1}{12} |m_{\text{Pl}}^2 - \xi \phi_0^2| \left(\frac{V'}{V} - 2\xi \frac{\phi_0}{m_{\text{Pl}}^2 - \xi \phi_0^2} \right)^2, \\ \eta_V &= \left| \frac{1}{12} (m_{\text{Pl}}^2 - \xi \phi_0^2) \left(\frac{V'}{V} + 2\xi \frac{\phi_0}{m_{\text{Pl}}^2 - \xi \phi_0^2} \right)^2 \right. \\ &\quad \left. + \frac{\xi}{3} - \frac{1}{6} \frac{V''}{V} (m_{\text{Pl}}^2 - \xi \phi_0^2) \right|. \end{aligned}$$

So, in the limit of cosmological constant at $\xi = -1$ we get

$$\epsilon_V = \frac{1}{3} \frac{\phi_0^2}{m_{\text{Pl}}^2 + \phi_0^2} \leq \frac{1}{3}, \quad \eta_V = \frac{1}{3} \frac{m_{\text{Pl}}^2}{m_{\text{Pl}}^2 + \phi_0^2} \leq \frac{1}{3},$$

therefore, the slow-roll approximation holds good. Furthermore, in this case

$$\frac{\dot{\phi}_0}{\phi_0} = H \quad \Rightarrow \quad \phi_0(t) = \phi_* a(t),$$

and the equation

$$H^2 = \frac{2V_0}{3m_{\text{Pl}}^2} \frac{1}{1 + a^2 \frac{\phi^2}{m_{\text{Pl}}^2}}$$

can be exactly integrated out, so that at small a we get the exponential inflation with a constant Hubble rate, $a \sim \exp(t\sqrt{2V_0/3m_{\text{Pl}}^2})$, while at large a the asymptotic behavior given in the previous subsection takes place. Thus, the vector field can provide a natural unification of the inflation and ordinary expansion, even at the simplest constant potential.

3.4. Matter and vector field

The conservation law for the matter with a constant parameter of state w

$$p_m = w\rho_m$$

results in

$$\rho_m = \frac{\rho_*}{a^n}, \quad n = 3(1 + w).$$

If the vector field has a trivial potential, i.e. a constant, then it again evolves as

$$\phi_0 \sim a,$$

so that at small a , when the matter dominates, we get a standard cosmology, while at large a we find a dynamical suppression of cosmological constant.

3.5. Interacting scalar and vector fields

It is an easy task to derive the evolution equations for the case of vector field interacting with a scalar field ϕ . We restrict ourselves by the consideration of slow-roll approximation in the case, when the interaction is given by a potential, only, without any dynamical terms, i.e. the derivatives of the fields. Moreover, we take the potential of the form

$$V(\phi_0, \phi) = U(\phi)(1 + 4\pi G \phi_0^2).$$

Then the slow rolling gives

$$H^2 = \frac{8\pi G}{3} U, \tag{39}$$

$$3H \dot{\phi}_0 - 3H^2 \phi_0 - 8\pi G \phi_0 = 0, \tag{40}$$

$$3H \dot{\phi} + \frac{\partial U}{\partial \phi} (1 + 4\pi G \phi_0^2) = 0, \tag{41}$$

where we put the signature for the vector field $\xi = -1$. The substitution of (39) into (40) leads to

$$\dot{\phi}_0 = 2H \phi_0 \quad \Rightarrow \quad \phi_0(t) = \phi_* a^2(t).$$

So, the vector field evolves quadratically with the scale factor. In the regime, when the vector field is small in comparison to the Planck mass, $4\pi G \phi_0^2 \ll 1$, the running of scalar quintessence and scale factor evidently repeats the situation with no any vector field. Therefore, we further consider the regime of

$$4\pi G \phi_0^2 \gg 1,$$

and specify the quintessence potential as a falling homogeneous function

$$U = U_0 \frac{M^{2n}}{\phi^{2n}}, \quad (42)$$

where M is a scale of mass, and n is a positive number. Further,

$$\dot{\phi} = H \phi', \quad \phi' = \frac{\partial \phi}{\partial \ln a}.$$

Then in the large vector field regime we get

$$2\phi\phi' = -\frac{\partial \ln U}{\partial \phi} \phi_*^2 a^4, \quad (43)$$

so that for the homogeneous potential we obtain

$$2\phi\phi' = 2n\phi_*^2 a^4,$$

that gives

$$\phi^2 = \frac{n}{2} \phi_*^2 a^4 + \phi_c^2,$$

where ϕ_c and ϕ_* are constants of integration. If ϕ_c dominates we have approximately got a constant scalar field and a constant Hubble rate of inflation. Otherwise, at $a \rightarrow \infty$ both fields proportionally evolve

$$\phi = \phi_0 \sqrt{n/2},$$

and

$$H^2 = U_0 \frac{8\pi G}{3} \frac{M^{2n}}{\phi_*^{2n}} \frac{1}{a^{4n}},$$

that can be easily integrated out,

$$a \sim t^{1/2n}, \quad H \sim \frac{1}{t}.$$

These relations can be compared with the standard quintessence evolution obtained by $\phi_0 = 0$, so that

$$\ln a \sim t^{2/(2+n)}, \quad H \sim t^{-n/(2+n)}.$$

Such the partnership of vector field with the scalar quintessence could be involved in the dynamics of universe today and in future, since a current cosmological constant

$$\Lambda = U_0 \frac{M^{2n}}{\phi_c^{2n}}$$

can be naturally small even at $U_0 \sim M^4$, $M \sim M_{\text{GUT}}$, if $\phi_c \gg M_{\text{GUT}}$ and $n \gg 1$, or at M giving the supersymmetry breaking scale. In addition, this “cosmological constant” will dynamically falling down in future.

4. Conclusion

In this paper we have derived generic equations for the evolution of flat homogeneous isotropic universe driven by a vector field.

We have found that the expansion induces a dynamical mass term for the vector field. Such the term can be treated as a source of running gravitational constant. This characteristic property can lead to a dynamical suppression of primary cosmological constant. The regimes of de Sitter expansion as well as a slowly rolling acceleration have been demonstrated. So, the vector field could serve not only as a quintessence partner, but it could compete with the scalar quintessence as an origin of dark energy in the universe.

Motivations for the consideration of vector field dynamics in the theory of gravitation could be manifold. So, as we have already mentioned, a success of scalar quintessence is significantly based on a freedom in a choice of scalar potentials, which can be motivated in some underlying theories. Nevertheless, there are several aspects, which seem to point to a way for a possible compelling extensions. First, a phantom quintessence modelled by a scalar field with a negative kinetic term can be *automatically* included in the consideration by introduction of non-gauge vector field as given by the lagrangian of ((1). Second, a present day quintessence has a characteristic scale of mass about the Hubble rate, while such the induced mass term is *automatically* generated by covariant derivatives for the non-gauge vector field. Third, scalar fields are involved in the explanation of flat rotation curves in dark galactic halos, so that the fields compose a triplet of monopole-like configuration [11] i.e. the spherically symmetric static field is proportional to the radius-vector: $\phi^a \propto \mathbf{x}$. This fact implies that explicit introduction of corresponding vector field could be promising in the aspect of dark matter in galaxies⁺. Next, if the vector field state at zero point $\phi^m \equiv 0$ is unstable, then the vector field could make the time arrow in the universe expansion.

We have considered several toy models, which have confirmed the physical effects expected due to the introduction of vector field as listed above. However, there are strong phenomenological constraints, which pose restrictions on a possible model. Indeed, the most strong constraints follow from the variation of Newton's constant and anomalous gravitational production of light fields during an inflation.

As for the variation of Newton's constant, we note that the vector field can cause a time-dependence, which is different from the variation of constant universality. So, the time-dependence is experimentally suppressed [20] at the limit of

$$\frac{|\dot{G}|}{G} \lesssim \text{few} \times 10^{-11} \text{ year}^{-1},$$

which is in agreement with a constraint following from the nucleosynthesis: the variation of Newton's constant at the time of nucleosynthesis in comparison with the present day value should be less than

$$\frac{\Delta G}{G} \lesssim 30\%,$$

so that dividing by the Universe age about 13.7 billion years, we get a similar estimate. Therefore, the corresponding evolution of temporal component ϕ_0 should be essentially suppressed. For instance, a preferable choice is a model with a vector field settled in a stable point, so that the evolution can be neglected, if the field mass is much greater than the Hubble rate.

⁺ This issue is considered in [19].

An analogous note has to be made on the vector field mass much greater than the Hubble rate, because of an anomalous gravitational production of fields with a similar mass scale as the vector field under consideration. Indeed, Felder, Kofman and Linde [21] have found that a moduli field coupled with the gravity can be copiously produced at early stages of inflation, if an effective mass of moduli has a dependence on the Hubble rate,

$$m_{\text{eff}}^2 = m^2 + c^2 H^2,$$

with a constant c and a ‘bare’ mass m . The production can be suppressed, if $c \gg 1$, or $H \ll 10^{14}$ GeV, or $m \gg H$. Then, preferably we should again adopt the constraint

$$m \gg H \quad (44)$$

during inflation, that implies that the vector field is, in practice, at rest with a mass about the Planck scale.

Thus, in cosmology a variety of vector field potential is actually restricted by the constraint of vector field stabilization with a scale about the Planck mass, while other models probably are removed since they lead to unacceptably large variation of Newton’s constant and anomalous gravitational production of light vector fields during the inflation.

In addition, to the moment there is no any fundamental physics framework for the introduction of extra vector fields in the phenomenology, to my knowledge. Nevertheless, one should point to that gauge vector fields of the Standard Model have longitudinal components, which are decoupled from the interactions with matter fields. The gauge invariance of interaction guarantees that the propagator of longitudinal component is not renormalized, i.e. it remains bare under the gauge interactions, and the longitudinal mode is never seen by the matter. In this respect the non-gauge lagrangian of (1) differs from the gauge invariant one by a specific gauge fixing term. This difference is never seen by the gauge-interacting matter. A natural question is whether this mode does play any role in the gravity or not (for an abelian field under consideration). So, the present paper can be useful in this aspect, too.

Phenomenological models are beyond the scope of this paper, so that the toy examples considered cannot answer a question whether the vector field could be a useful partner of quintessence in practice or not, but we have shown that the vector field could be the partner in principle. Advantages of vector field partnership with the quintessence could be model-dependent.

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